4.8 Plane Parallel Plates

Figure 4.12 shows a lens which, in air, would form an image at P. The insertion of the plane parallel plate between the lens and P displaces the image to P'. If we trace the path of the light rays through the plate, we first notice that the ray emerging from the plate has exactly the same slope angle that it had before passing through the plate, since by Snell's Law, $\sin I_1' = (1/N)\sin I_1$, and $I_2 = I_1'$ (since the surfaces are parallel). Thus, $\sin I_2 = \sin I_1' = (1/N)\sin I_1 = (1/N)\sin I_2'$, and $I_1 = I_2'$. Therefore, the effective focal length of the lens system, and the size of the image, are unchanged by the insertion of the plate.

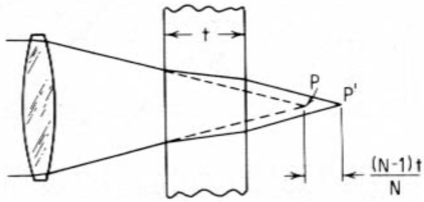

FIG. 4.12. The longitudinal displacement of an image by a plane parallel glass plate.

The amount of longitudinal displacement of the image is readily determined by application of the paraxial ray tracing formulae of Chapter 2, and is equal to $(N-1)t/N$. The effective thickness of the plate compared to air (the equivalent air thickness) is less than the actual thickness t by the amount of this shift. The equivalent air thickness is thus found by subtracting the displacement from the thickness and is equal to t/N. The concept of equivalent thickness is useful when one wishes to determine whether a certain size prism can be fitted into the available air space of an optical system, and also in prism system design.

If the plate is rotated through an angle I as shown in Fig. 4.13, it can be seen that the "axis ray" is laterally displaced by an amount D, which is given by

$$D = t\cos I(\tan I - \tan I')$$

or

$$D = t\sin I\left[1 - \sqrt{\frac{1-\sin^2 I}{N^2 - \sin^2 I}}\right]$$

For small angles, we can make the usual substitution of the angle for its sine or tangent to get

$$d = \frac{tI(N-1)}{N}$$

This lateral displacement of a tilted plate is made use of in high speed cameras (where the rotating plate displaces the image an amount approximately equal to the travel of the continuously moving film) and in optical micrometers. The optical micrometer is usually placed in front of a telescope and used to displace the line of sight. The amount of displacement is read off a calibrated drum connected to the mechanism which tilts the plate.

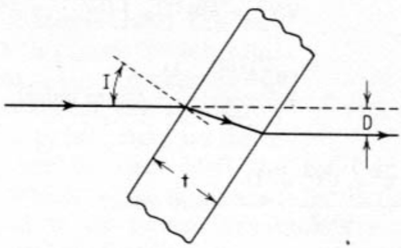

FIG. 4.13. The lateral displacement of a ray by a tilted plane parallel plate.

When used in parallel light, a plane parallel plate is free of aberrations (since the rays enter and leave at the same angles). However, if the plate is inserted in a convergent or divergent beam, it does introduce aberrations. The longitudinal image displacement $(N - 1)t/N$ is greater for short wavelength light (higher index) than for long, so that overcorrected chromatic aberration is introduced. The amount of displacement is also greater for rays making large angles with the axis; this is, of course, overcorrected spherical aberration. When the plate is tilted, the image formed by the meridional rays is shifted backward while the image formed by the sagittal rays (in a plane perpendicular to the page in the figures) is not, so that astigmatism is introduced.

The amount of aberration introduced by a plane parallel plate can be computed by the formulae below. Reference to Fig. 4.14 will indicate the meanings of the symbols

U and u – slope angle of the ray to the axis

U_p and u_p – the tilt of the plate

t – thickness of the plate

N – index of the plate

V – Abbe V number $(N_D - 1)/(N_F - N_C)$

$$\text{Chromatic Aberration} = l'_F - l'_C = \frac{t(N-1)}{N^2 V}$$

$$\text{Spherical Aberration} = L' - l' = \frac{t}{N}\left[1 - \frac{N\cos U}{\sqrt{N^2 - \sin^2 U}}\right] \text{ (exact)}$$

$$= \frac{tu^2(N^2 - 1)}{2N^3} \text{ (third order)}$$

$$\text{Astigmatism} = l'_s - l'_t = \frac{t}{\sqrt{N^2 - \sin^2 U_p}}\left[\frac{N^2\cos^2 U_p}{(N^2 - \sin^2 U_p)} - 1\right] \text{ (exact)}$$

$$= \frac{tu_p^2(N^2 - 1)}{N^3} \text{ (third order)}$$

$$\text{Sagittal Coma} = \frac{tu^2u_p(N^2 - 1)}{2N^3} \text{ (third order)}$$

$$\text{Lateral chromatic} = \frac{tu_p(N - 1)}{N^2V} \text{ (third order)}$$

These expressions are extremely useful in estimating the effect that the introduction of a plate or a prism system will have on the state of correction of an optical system.

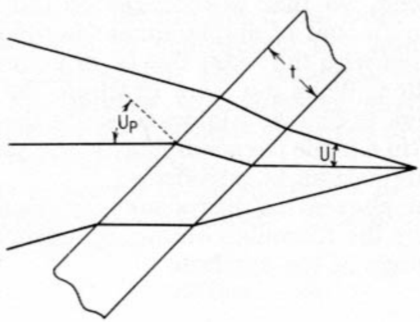

FIG. 4.14.

4.9 The Right Angle Prism

The right angle prism, with angles of 45°-90°-45°, is the building block of most non-dispersing prism systems. Figure 4.15 shows a parallel bundle of rays passing through such a prism, entering through one face, reflecting from the hypotenuse face and leaving through the second face. If the rays are normally incident