SNR tests vs F/#
ML4022, Pentax 67 400mm F/4 EDIF

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This document is updated frequently
always use the latest version:

http://www.narrowbandimaging.com/incoming/Fnumber_and_SNR_with_ML4022_binned_DTC_PTC_latest_version.pdf

Purpose/ Approach

- Answer the question: does F/# affect SNR?
- This work uses Photon Transfer methods to analyze real empirical data.
  - The analysis method is clearly explained
  - A real camera and test target are used
  - Images are taken at varying f/# settings and exposure times
  - All images are flat-fielded prior to analysis
  - A modulation PTC is created
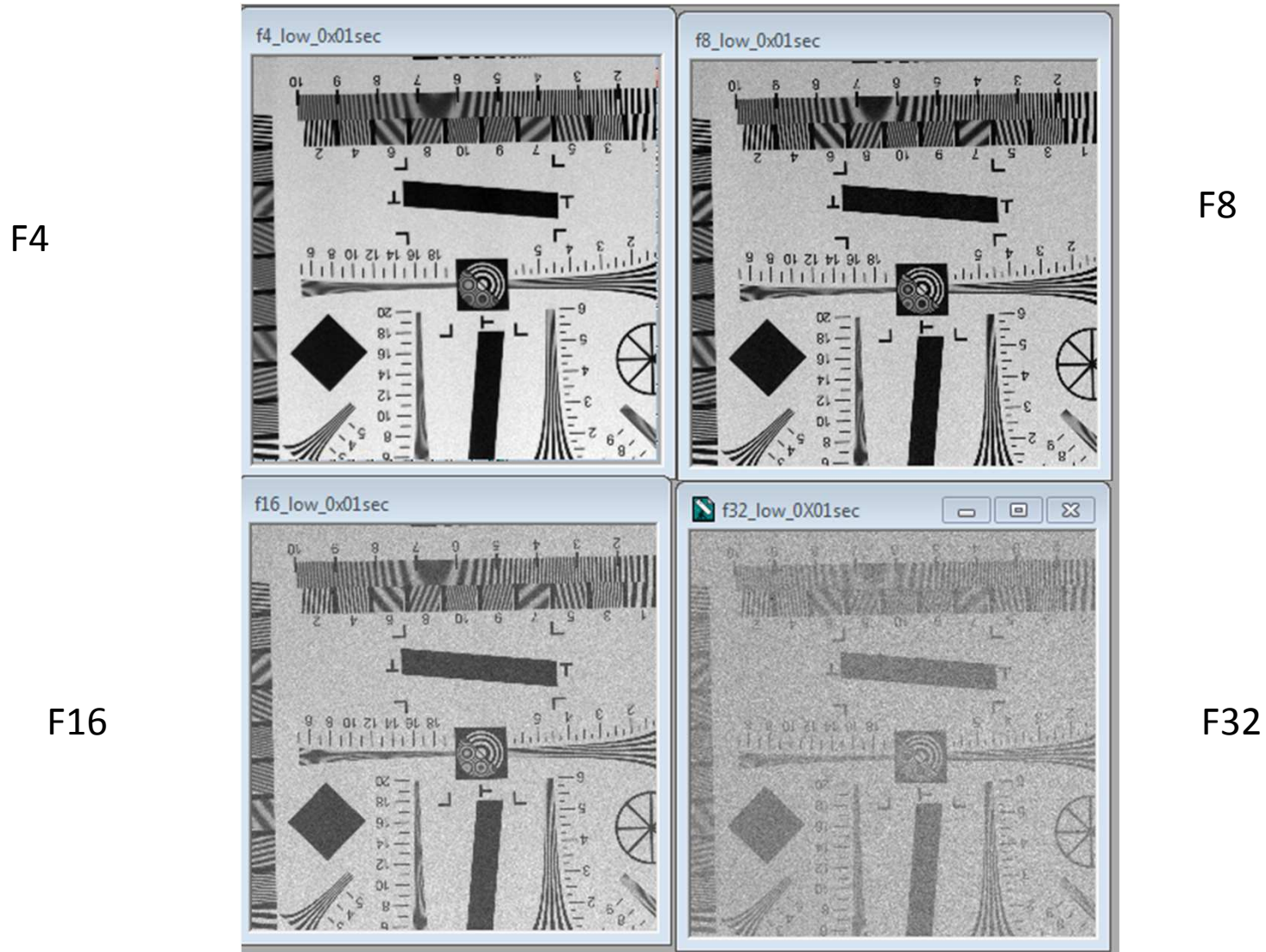 - Families of curves are created for exposure time, F/# and SNR

Equipment

ML4022 with Pentax 67
400mm f/4 EDIF

(foil used to prevent light leaks)
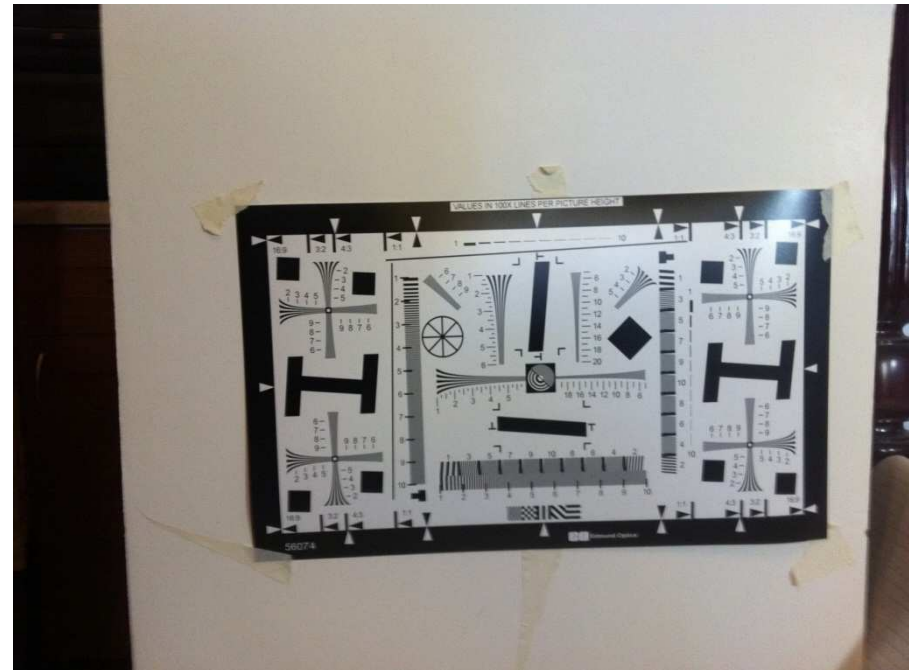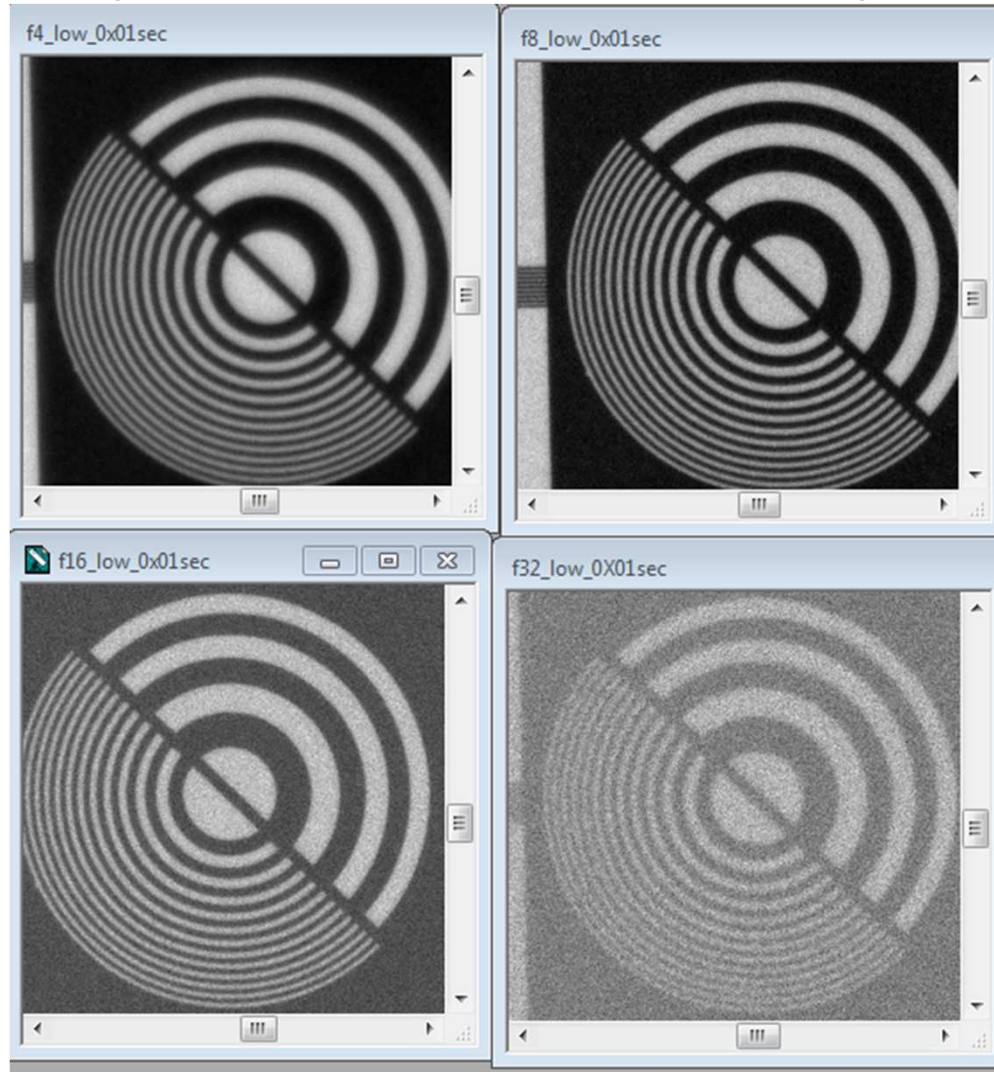

Standard Resolution Test Target

Example: 0.1 second exposures (full FOV)

All images fully calibrated (flat-fielded and despiked)

Example: 0.1 second exposures (zoomed to 100%)

F4

F8

F16

F32

All images fully calibrated

Analysis of Images with Modulation

An image with modulation can be analyzed using the noise equation, by treating the RMS modulation as another noise term:

$$\delta_{total} = \text{SQRT}(\delta_{modulation}^2 + Read\_noise^2 + Shot\_noise^2 + fixed\_pattern\_noise^2 + Dark\_fixed\_pattern\_noise^2 + Dark\_shot\_noise^2)$$

If the image to be analyzed has been flat-fielded and the operating temperature is low enough, then the dark and fixed pattern noise terms can be ignored simplifying the equation to:

$$\delta_{total} = \text{SQRT}(\delta_{modulation}^2 + Read\_noise^2 + Shot\_noise^2)$$

Images with Modulation

Mathematically the modulation, $\delta_{modulation}$, is modeled the same as fixed pattern noise, hence the value of the modulation is proportional to the average signal level. In this case instead of PRNU for FPN analysis a different constant, M_I is used.

For the analysis, the Modulation, $\delta_{modulation}$, is decomposed into a modulation constant, M_I, and an equivalent flat-field image with a signal level equal to the average value of the modulated image. The modulation constant, M_I , can be thought of like contrast.

$$\delta_{modulation} = M_I * \text{Equivalent\_flatfield\_average\_signal}$$

In general each feature in the image will have a different M_I and it is the variation of the M_I across the image that gives the image its appearance.

Measuring Signal to Noise from Images

To calculate the signal to noise ratio of a modulated image you begin with the noise equation and solve for M_I where the total modulation, δ_{total}, is what is measured by using the Standard Deviation measuring tool on sampling unit (100 x 100 pixel in this analysis) measurement box placed over a region of interest in the image. The image must have the offset removed. If the image is fully calibrated (flat-fielded and despiked), the analysis is simplest because the FPN and dark signal terms can be ignored and the offset is removed.

To solve the equation you need to know a few parameters:

Read noise:
Using Photon Transfer analysis, the read noise of the camera can be accurately measured.
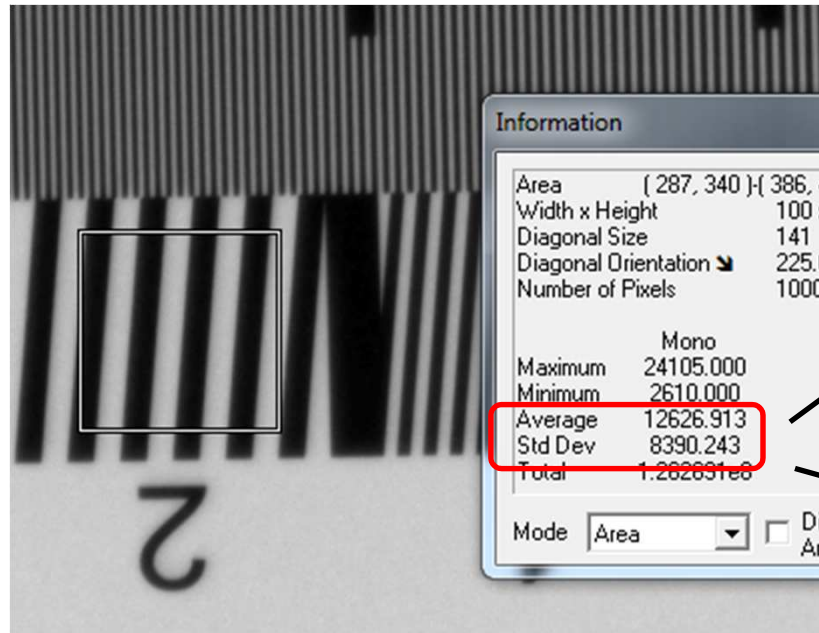
Total RMS modulation ($\delta_{modulation}$):
This is the standard deviation measured in the selection box when placed over a region of interest in the image

Average Signal Level:
This is measured at the same time as the Standard Deviation. This will be the signal level for the equivalent flat-field image and will be used to calculate the shot noise of the equivalent flat-field image

Example of Image Measurements needed to calculate SNR

Information

Area	(287, 340)-(386, 4
Width x Height	100 x
Diagonal Size	141
Diagonal Orientation ↘	225.0
Number of Pixels	1000

	Mono
Maximum	24105.000
Minimum	2610.000
Average	12626.913
Std Dev	8390.243
Total	1.262631e8

Mode Area ▼ ☐ Di: Ar

Average = Flat-Field average signal value

Signal_shot_noise =
SQRT(Flat-Field average signal)

Standard Deviation = δ_{total}

Example:
Assume Read Noise = 10.1DN

δ_{total}= 8390.243 = SQRT((M_I*Flat-Field_average_signal)^2+ read_noise^2+
Flat-field_average_signal)

Solving for M_I we get: M_I= 66.441%

Calculating the SNR

The SNR of a modulated image is calculated by using the M_I factor multiplied by the SNR of a flat field image of the same average signal level:

$$SNR_{modulated\ image} = M_I * SNR_{Equivalent\ FF}$$

The total noise of the equivalent flat-field is equal to the shot noise of the flat field of that signal level: Noise = SQRT(Flat-Field Signal level)
SNR = (Flat-Field Signal level)/SQRT(Flat-Field Signal level)

SNR = SQRT(Flat-Field Signal level)

So in our example, the Signal Level for the Equivalent Flat-Field was 12,626.913DN, making the SNR of the equivalent flat-field:
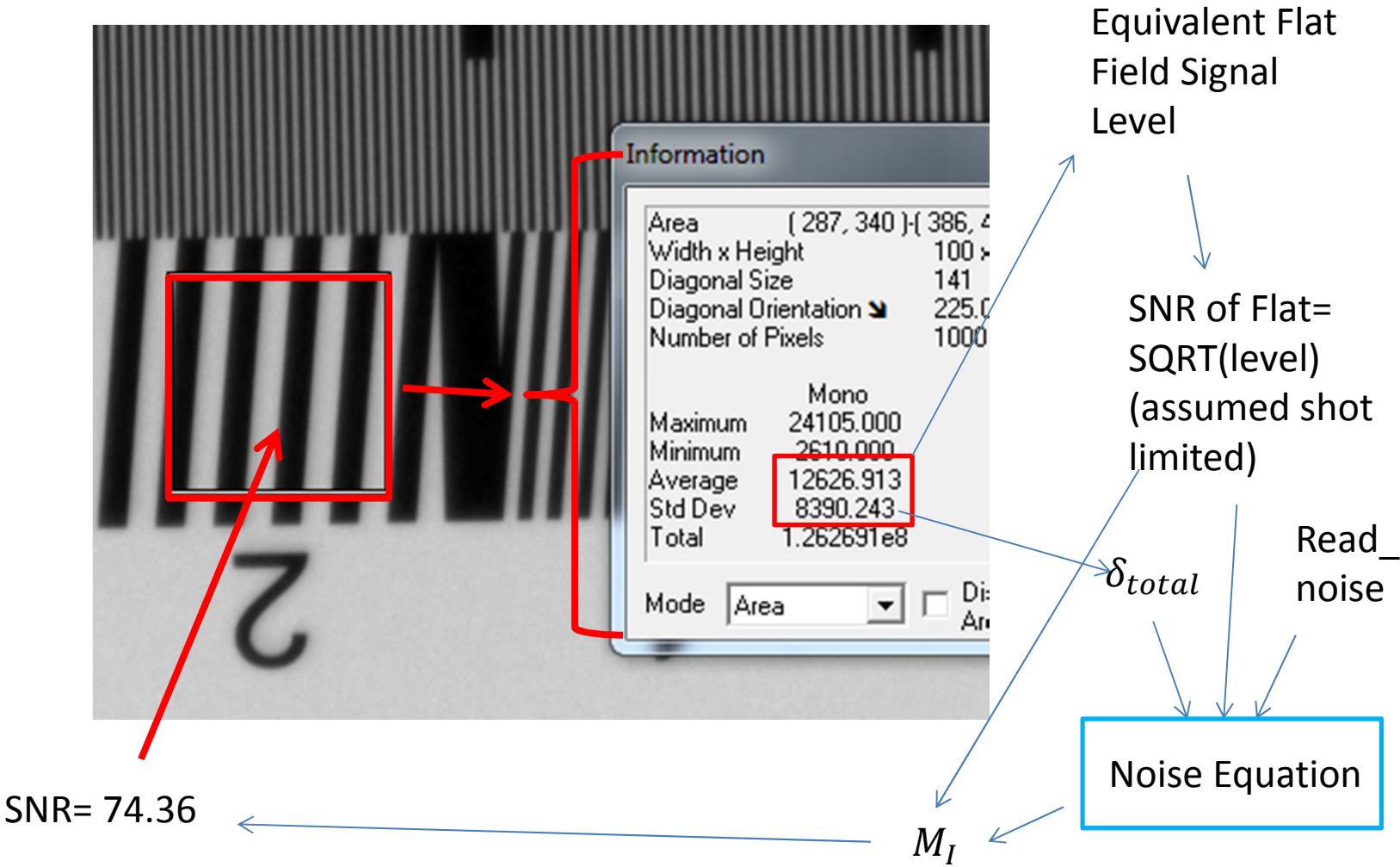
SQRT(12,626.913) = 112.369

So with an M_I = 66.441% we get a final SNR for the modulated image region of:
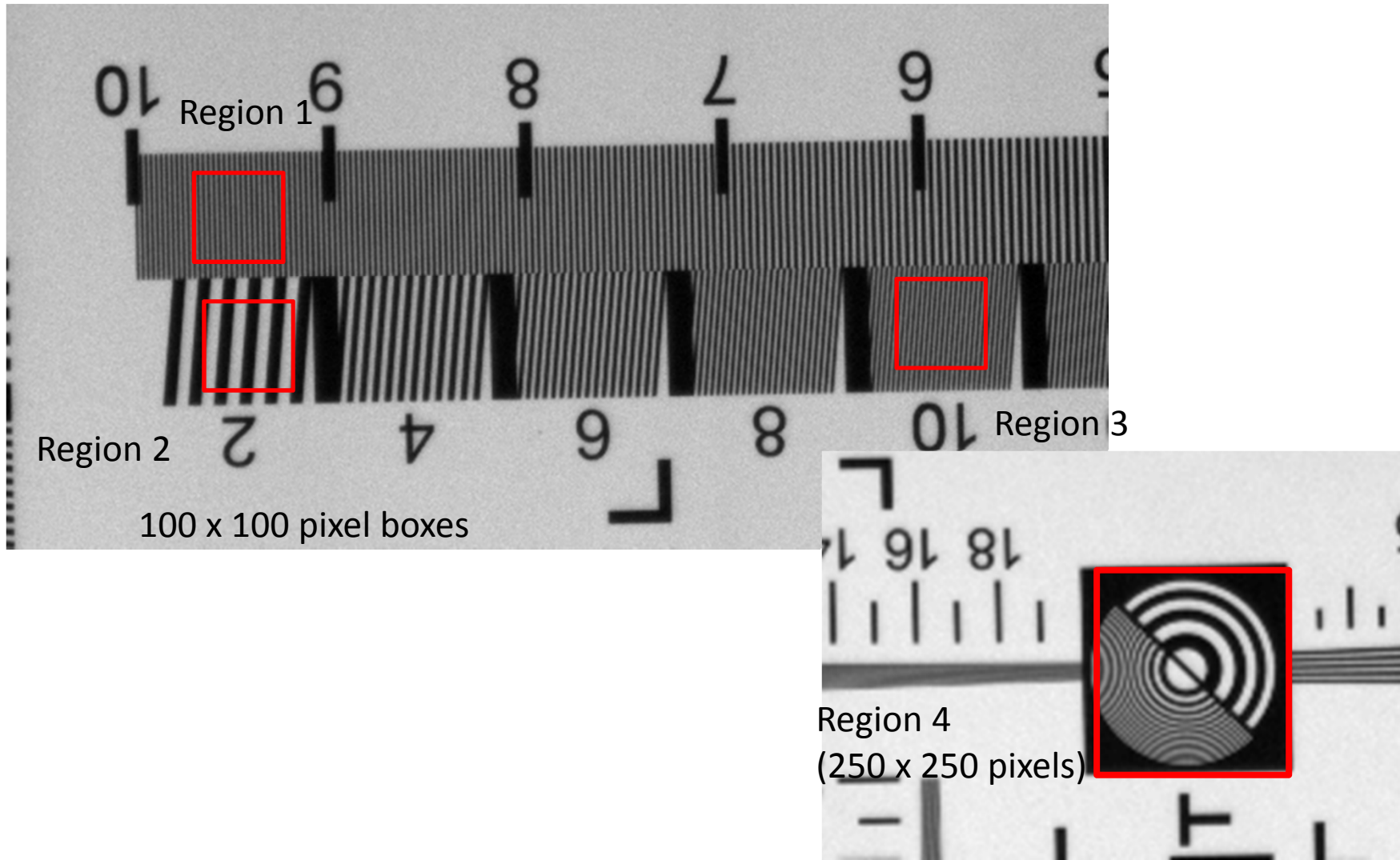$$SNR_{image-region} = 0.66441* 112.369 = 74.36$$

This is the calculation method that will be used for creating the curves that follow

Parameter Sources and Interrelationships

Sampled Regions used for Analysis

All images fully calibrated

Spreadsheet used for creating curves

	4 F#												
time(s)	region1 sigma	region 1 average	region 2 sigma	region 2 average	region 3 sigma	region 3 average	read noise (dn)	M1	M2	M3	SNR1	SNR2	SNR3
0.01	309.792	1306.083	830.733	1441.364	340.763	1323.153	10.1	0.23545	0.57571	0.25595	8.19492	21.1222	8.97095
0.05	1525.554	5665.475	4162.332	6337.25	1667.268	5718.565	10.1	0.26894	0.65668	0.29125	20.063	51.8607	21.8307
0.1	3280.457	11105.738	8390.243	12626.913	3392.997	11199.824	10.1	0.29523	0.66441	0.3028	30.9705	74.36	31.9004
0.12	4013.682	13205.32	9966.422	14730.062	4204.7	13345.88	10.1	0.30382	0.67655	0.31494	34.7791	81.8288	36.2445

	8 F#			112.3695377									
time(s)	region1 sigma	region 1 average	region 2 sigma	region 2 average	region 3 sigma	region 3 average	read noise (dn)	M1	M2	M3	SNR1	SNR2	SNR3
0.01	151.876	475.352	301.553	515.578	160.97	478.418	10.1	0.31548	0.58289	0.33267	6.24109	12.093	6.60619
0.04	636.796	1502.811	1228.095	1694.735	664.258	1508.258	10.1	0.4229	0.72422	0.43961	15.8645	28.9554	16.5232
0.05	718.685	1856.207	1523.54	2111.539	802.221	1855.669	10.1	0.33971	0.72119	0.43165	14.2499	32.367	18.1035
0.1	1468.962	3575.503	3008.874	3979.114	1601.686	3568.816	10.1	0.41049	0.756	0.44848	24.2027	47.0886	26.4171
0.12	1813.459	4261.403	3636.043	4783.054	1921.571	4256.485	10.1	0.42527	0.76005	0.45118	27.4351	52.0132	29.0892
0.2	3071.724	6986.186	6010.199	7781.884	3239.304	6977.137	10.1	0.43952	0.77225	0.46412	36.4713	67.6817	38.4871
0.4	5925.252	13690.644	11846.828	15287.829	6295.302	13723.667	10.1	0.43271	0.77488	0.45864	50.4426	95.4907	53.53

	16 F#												
time(s)	region1 sigma	region 1 average	region 2 sigma	region 2 average	region 3 sigma	region 3 average	read noise (dn)	M1	M2	M3	SNR1	SNR2	SNR3
0.01	41.664	188.501	75.02	198.525	44.063	192.249	10.1	0.20169	0.36766	0.21112	2.23054	4.2103	2.36603
0.05	197.829	535.508	377.347	589.631	209.777	549.577	10.1	0.3664	0.63842	0.37887	7.77101	14.3134	8.15704
0.1	399.885	971.796	757.943	1082.665	418.869	998.98	10.1	0.36811	0.69935	0.41798	10.9167	21.9983	12.584
0.16	634.216	1488.433	1203.743	1650.101	667.155	1530.766	10.1	0.42525	0.72906	0.43503	15.8715	28.7402	16.4803
0.8	2332.646	7019.818	5842.041	7825.343	2367.422	7235.641	10.1	0.33208	0.74647	0.32697	27.6229	65.607	27.6193
1.6	6191.061	13994.171	11892.156	15586.49	6569.473	14384.459	10.1	0.44232	0.76294	0.45663	52.1356	94.9392	54.5728
1.92	7355.784	16380.18	13945.175	17997.175	7778.309	16937.145	10.1	0.449	0.77482	0.45918	57.2869	103.651	59.5799

	32 F#												
time(s)	region1 sigma	region 1 average	region 2 sigma	region 2 average	region 3 sigma	region 3 average	read noise (dn)	M1	M2	M3	SNR1	SNR2	SNR3
0.01	13.538	120.721	20.782	123.868	13.798	121.798	10.1	#NUM!	0.11587	#NUM!	#NUM!	0.95502	#NUM!
0.05	37.38	206.931	89.583	220.784	37.321	212.618	10.1	0.17392	0.39751	0.15444	2.04757	4.88481	1.85121
0.1	70.883	317.608	179.931	347.273	71.052	329.202	10.1	0.1954	0.51452	0.20641	3.02969	8.42969	3.27222
0.12	82.46	361.257	213.124	392.65	82.698	375.636	10.1	0.22035	0.53982	0.21233	3.69833	9.53018	3.64939
0.64	444.433	1512.397	1153.264	1708.825	440.223	1585.905	10.1	0.29266	0.67443	0.27637	11.0158	27.0828	10.6684
3.2	2074.488	6938.124	5583.935	7844.337	2099.273	7389.473	10.1	0.29875	0.71175	0.28385	24.7039	62.6327	24.2335
6.4	3958.848	13681.076	10916.267	15187.218	3999.437	14418.11	10.1	0.28924	0.71873	0.27726	33.7058	88.2782	33.1754
7.68	5154.991	17002.033	13380.524	18844.08	5112.526	17858.443	10.1	0.3031	0.71003	0.28618	39.4039	97.2054	38.1353

Modulation PTC (MPTC)
Modulation, Noise and SNR vs Signal Level

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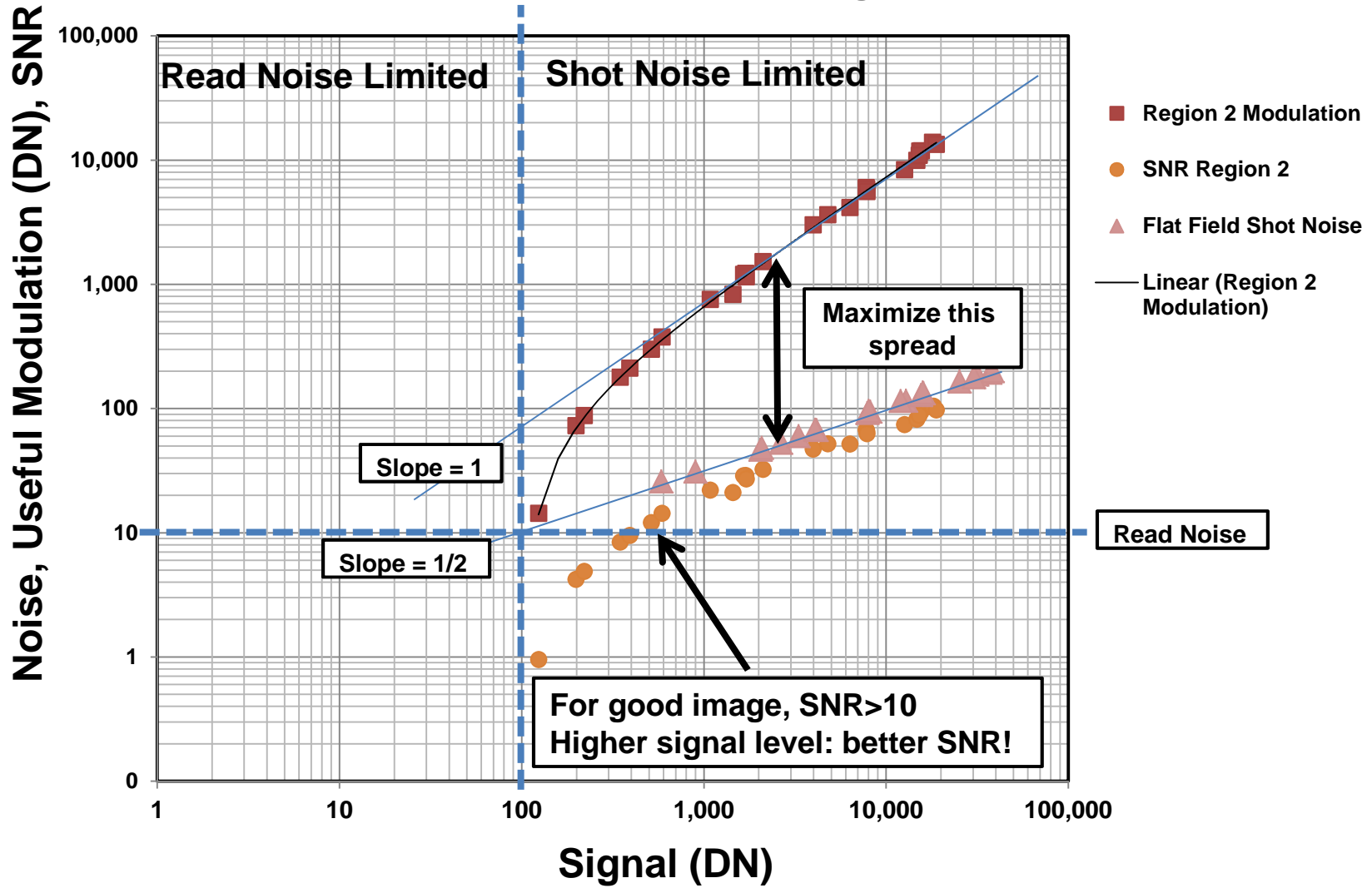
What does that Chart Mean?

- The higher the average signal level the:
  - Higher the SNR
  - Higher the useful modulation
- With a read noise of 10DN, you need at least 100DN of signal to have the shot noise exceed the read noise: more signal is better….
- To have a decent image, SNR should be greater than 10
- You need to have at least 500DN worth of signal to reach that point in Region 2 of the image. More signal: better SNR….
- ***The question is how much time does it take to reach a certain SNR for a given optical configuration***

SNR vs Time vs F/#

SNR vs Time vs F/#
ML4022, Pentax 67 400mm EDIF
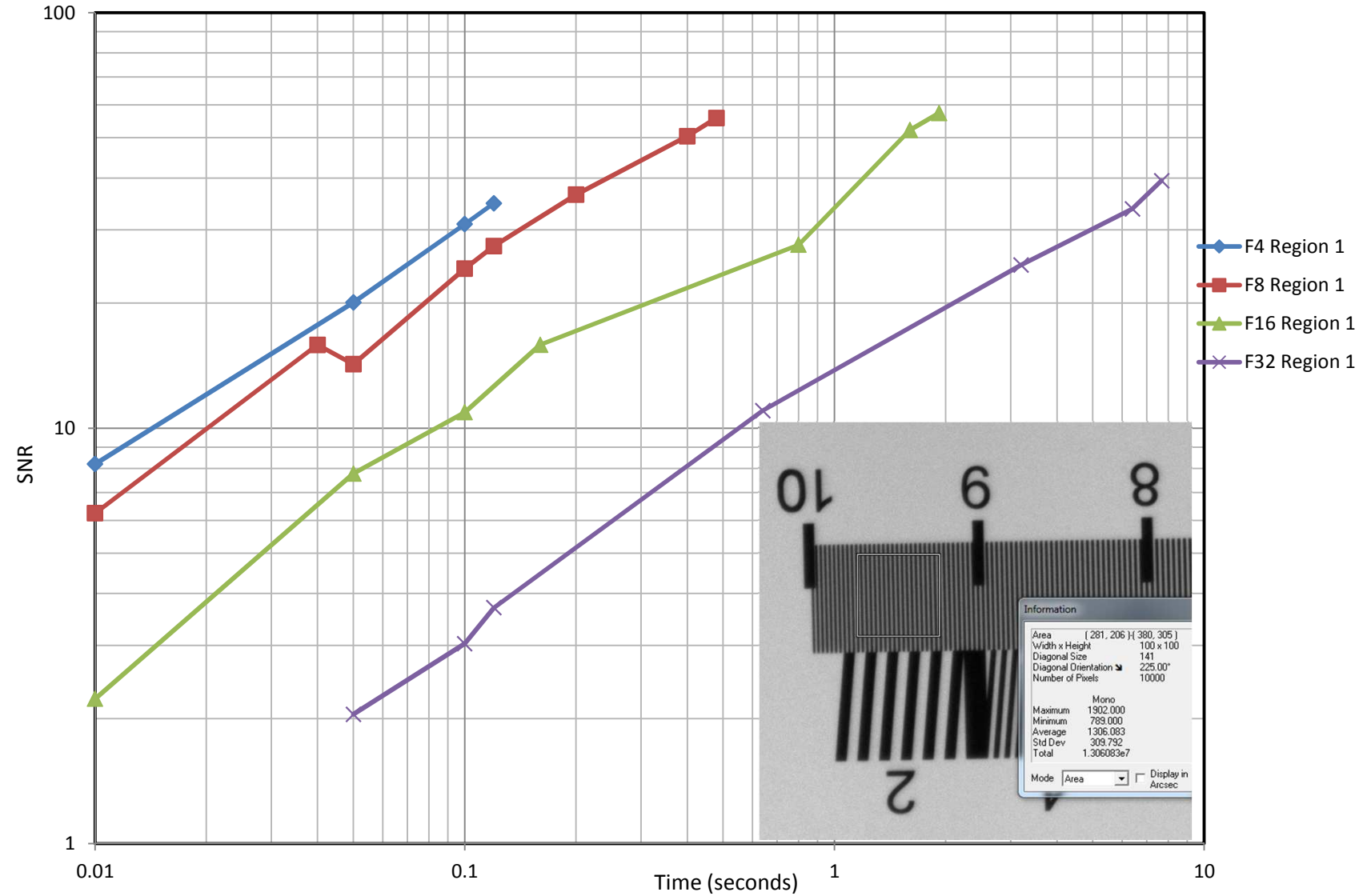
All images fully calibrated

SNR vs Time vs F/#
ML4022, Pentax 67 400mm EDIF
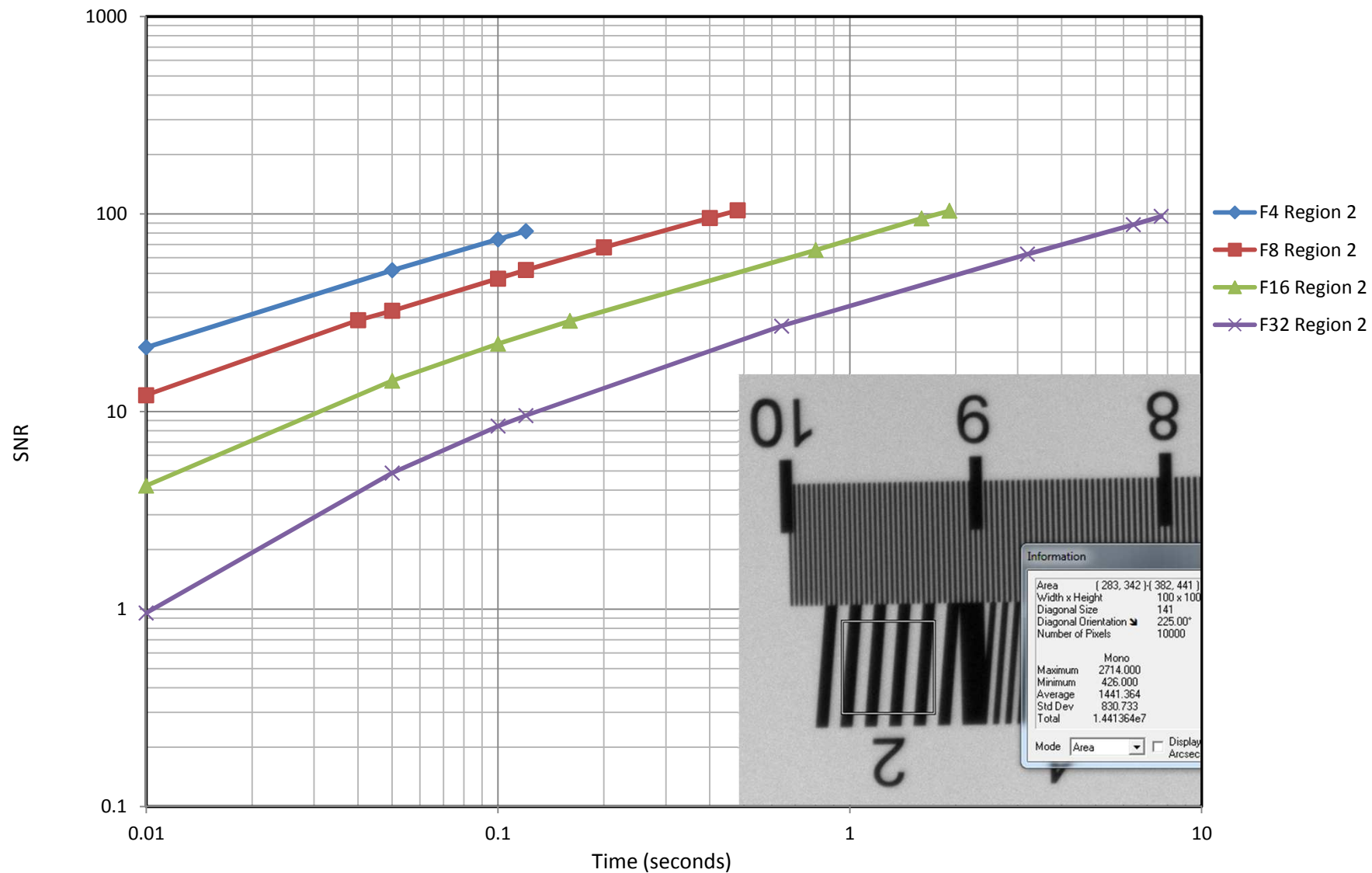
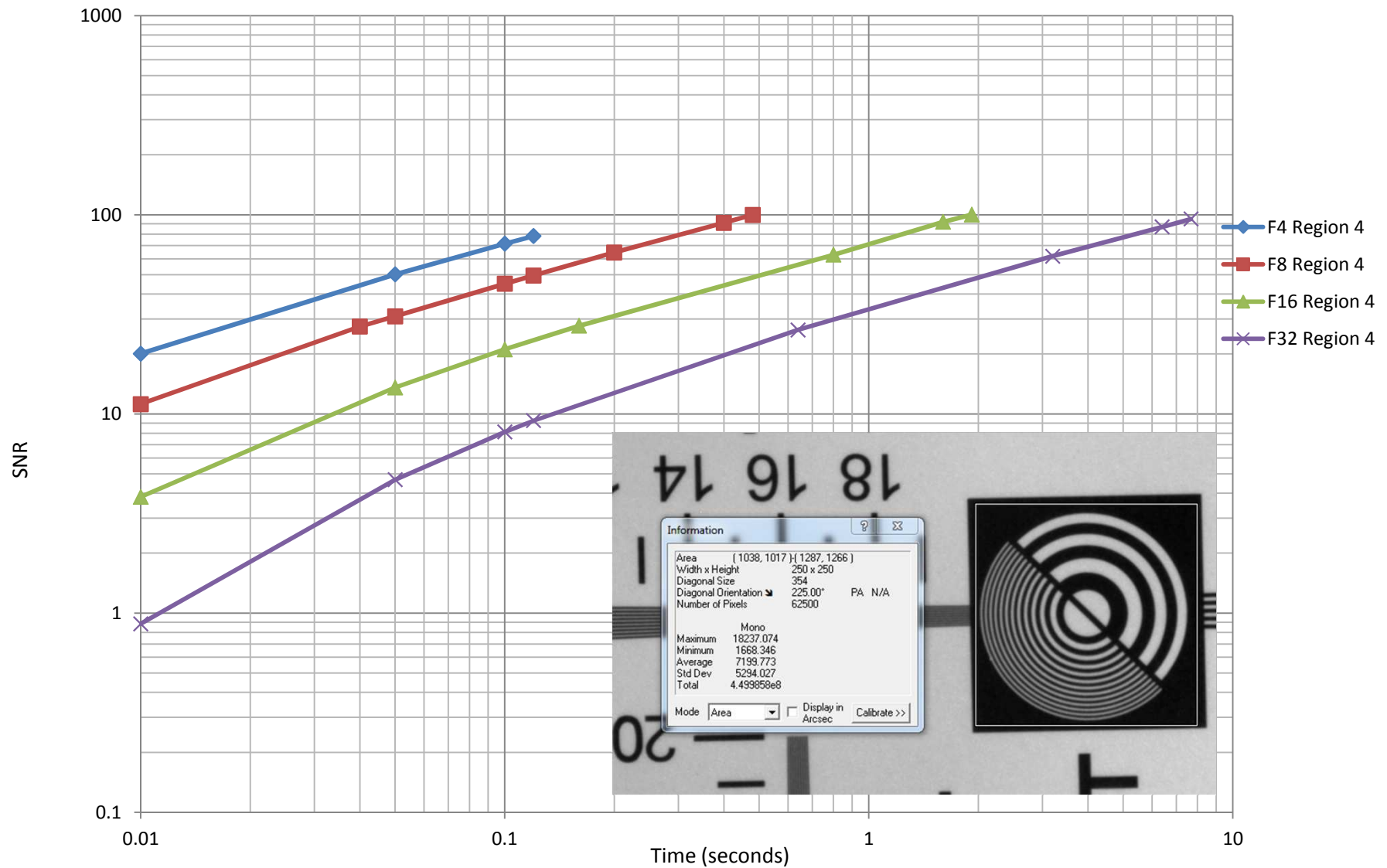All images fully calibrated

SNR vs Time vs F/#
ML4022, Pentax 67 400mm EDIF
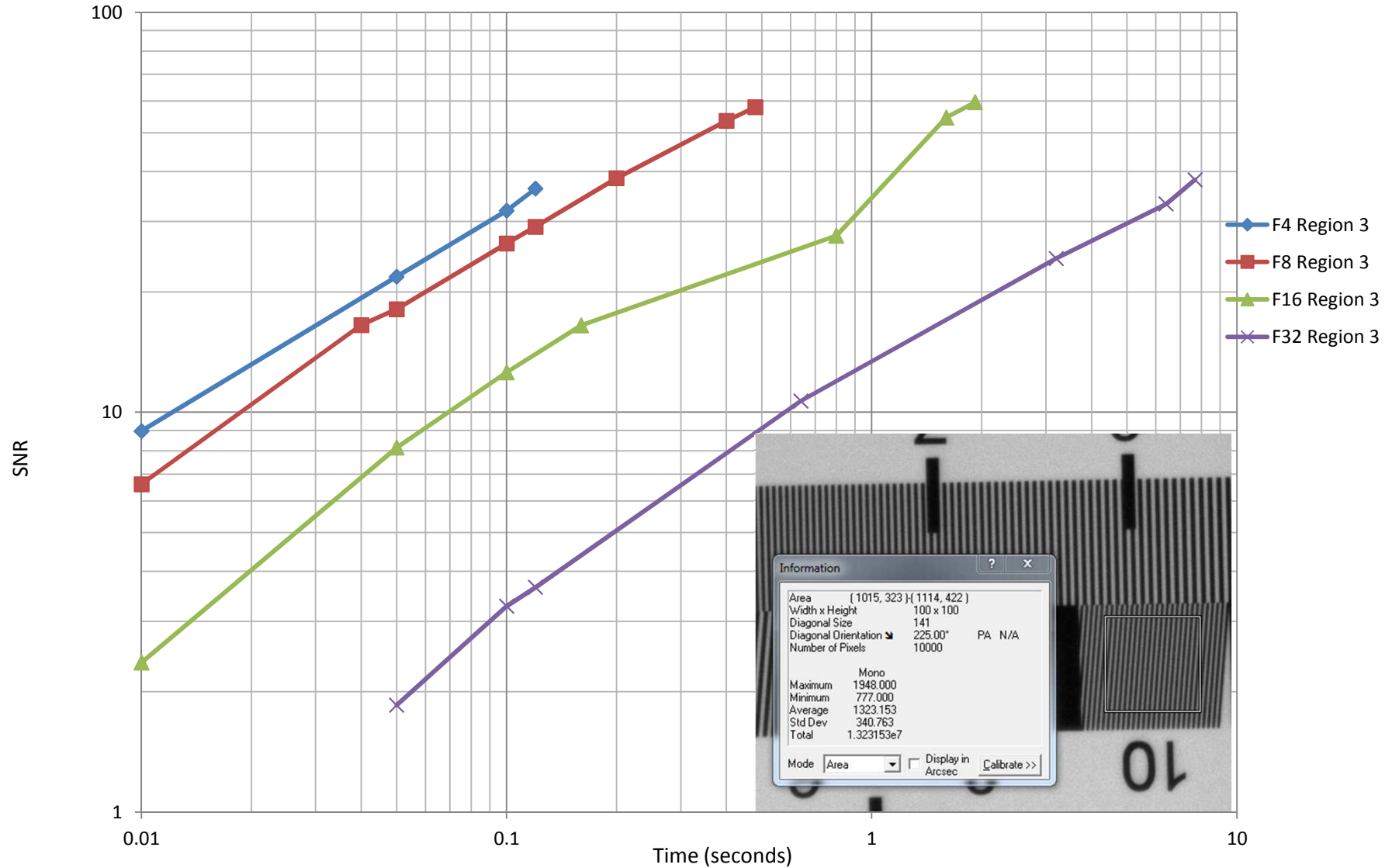
All images fully calibrated

SNR vs Time vs F/#
## ML4022, Pentax 67 400mm EDIF

All images fully calibrated

Key Observations

- The F/# strongly affects the SNR. For a doubling of F/# (ie F/4 -> F/8), there is a quadrupling of exposure time needed to reach the same SNR once the signal is shot noise limited: (sqrt(signal)> read_noise)

- The F/4 setting of the lens appears to actually be a bit "slower"... the ratios among f/8, f/16 and f/32 match the theory well but F/4 seems a bit slower than advertised

Equivalent Exposures (full FOV)

F4
0.01 seconds
Avg Signal Level
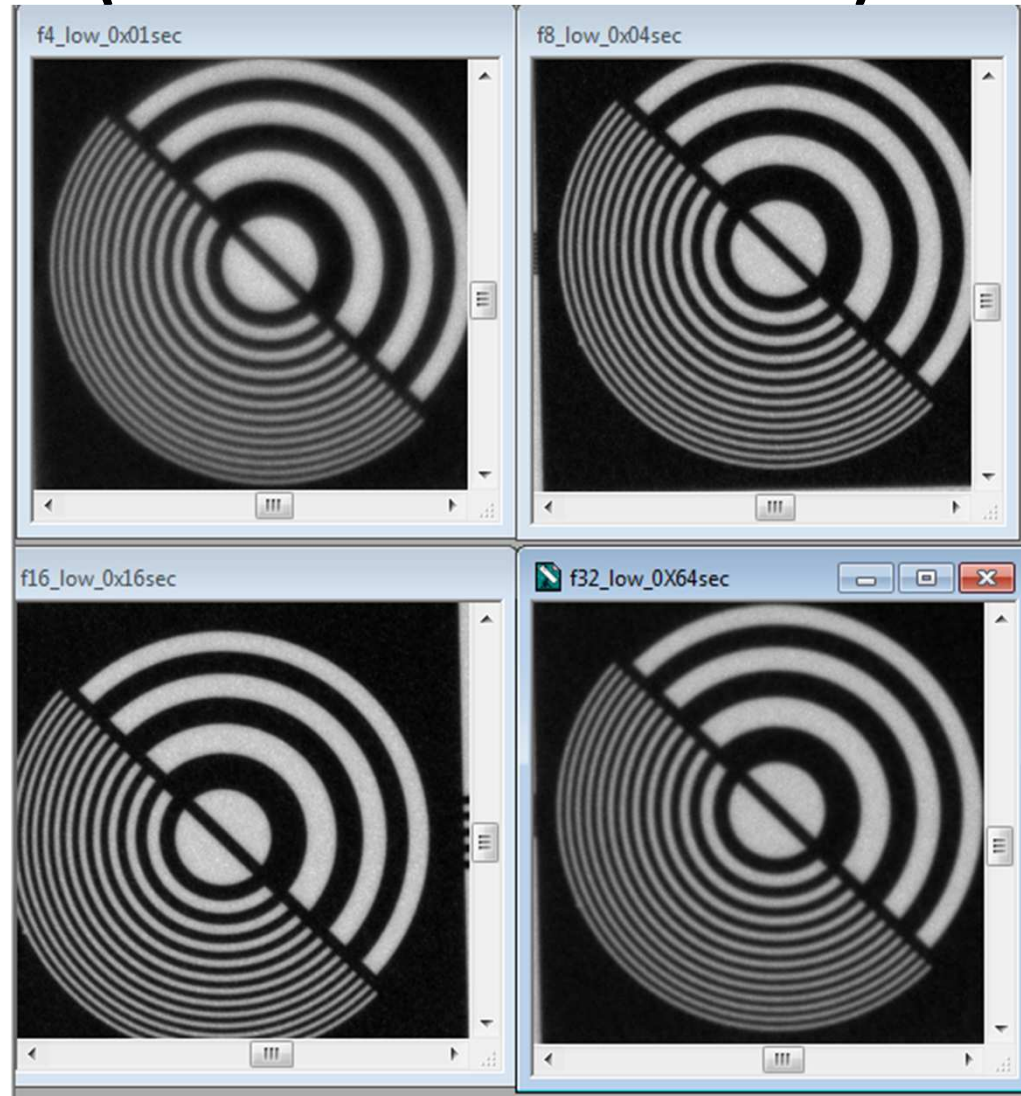2352DN
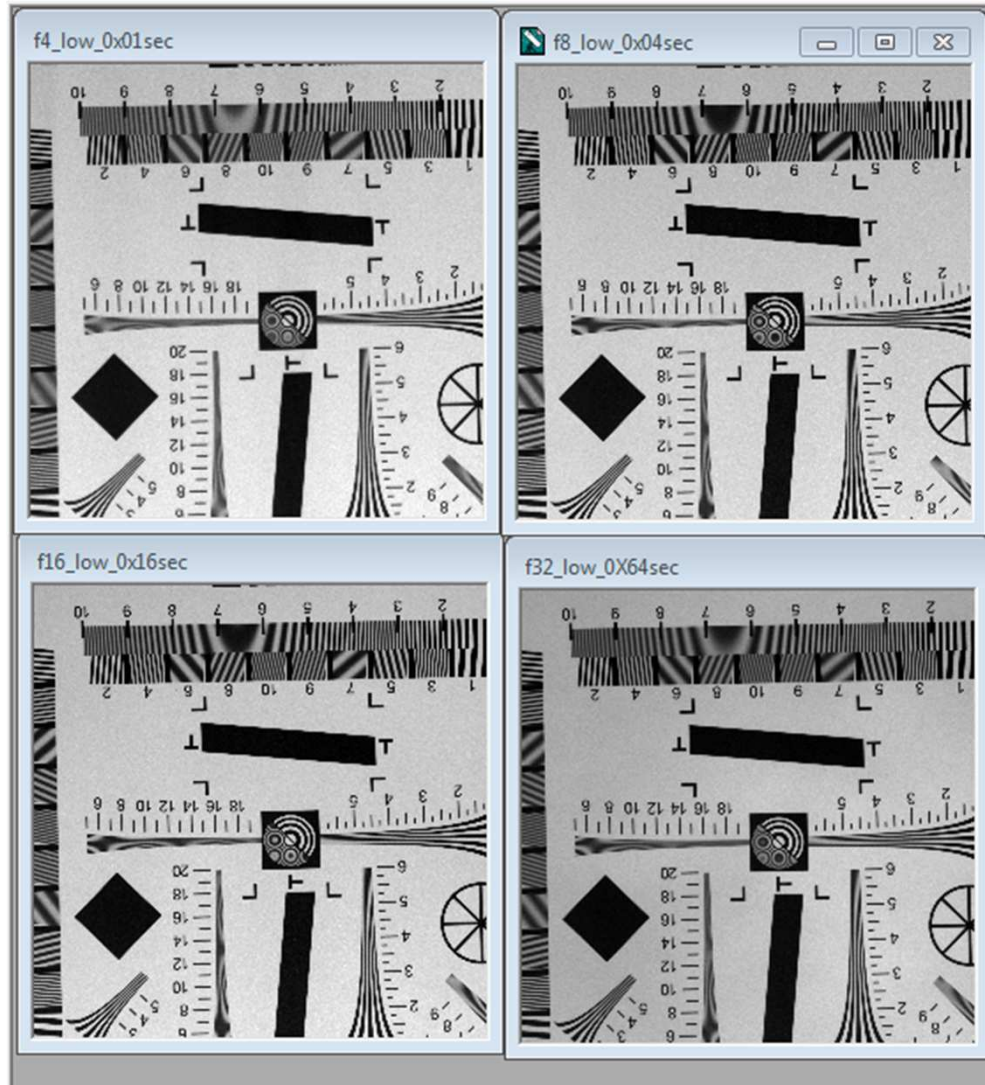
F8
0.04 seconds
Avg Signal Level
2760DN

F16
0.16 seconds
Avg Signal Level
2748DN

F32
0.64 seconds
Avg Signal Level
2840DN

All images fully calibrated

Equivalent Exposures
(zoomed to 100%)

F4
0.01 seconds
Avg Signal Level
2352DN

F8
0.04 seconds
Avg Signal Level
2760DN

F16
0.16 seconds
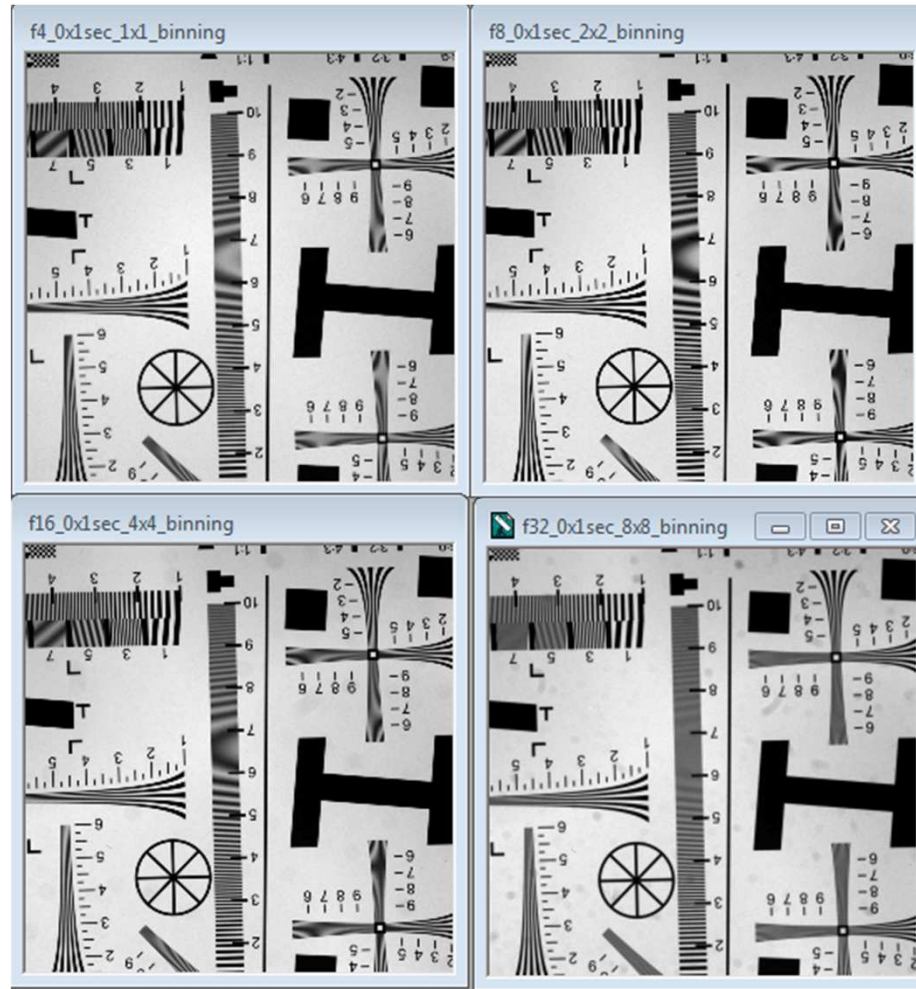Avg Signal Level
2748DN

F32
0.64 seconds
Avg Signal Level
2840DN

What about Binning?

- On-chip binning (summing) of pixels can compensate for changing F/#

- A doubling of F/# (ie F/4->F/8) would otherwise require a quadrupling of exposure time, but 2x2 binning gives the same amount of charge in the same time by collecting charge in four pixels

- Binning can also increase the effective well capacity depending on the sensor and camera design in question: this can permit deeper exposures to be made and in less time than unbinned (see the ML4022 PTC report appended to the end of this report)

Binning for exposure time compensation required by F/# change

F4
0.01 seconds
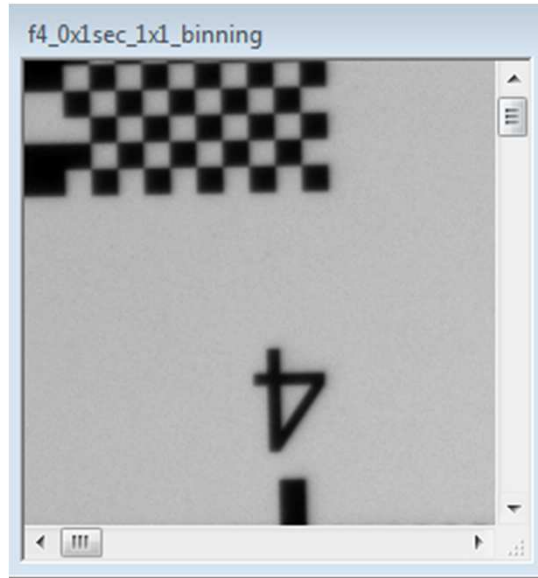Avg Signal Level
14,066 DN
1x1 binning

F8
0.01 seconds
Avg Signal Level
17358DN
2x2 binning

F16
0.01 seconds
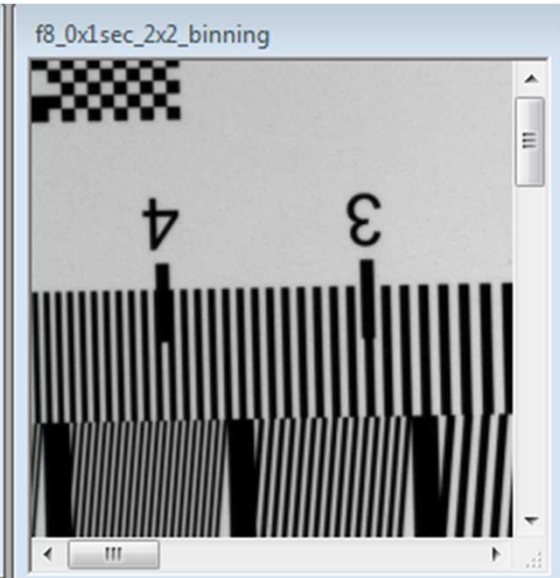Avg Signal Level
17315DN
4x4 binning

F32
0.01 seconds
Avg Signal Level
18054DN
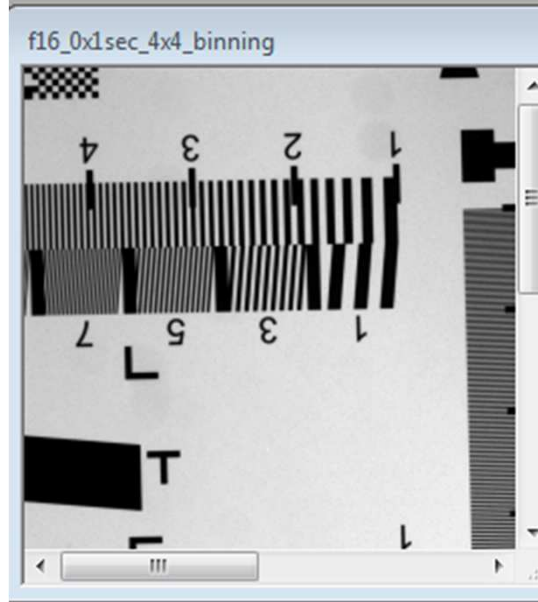8x8 binning

The Ugly Side of Binning: Resolution Loss

F4
0.01 seconds
Avg Signal Level
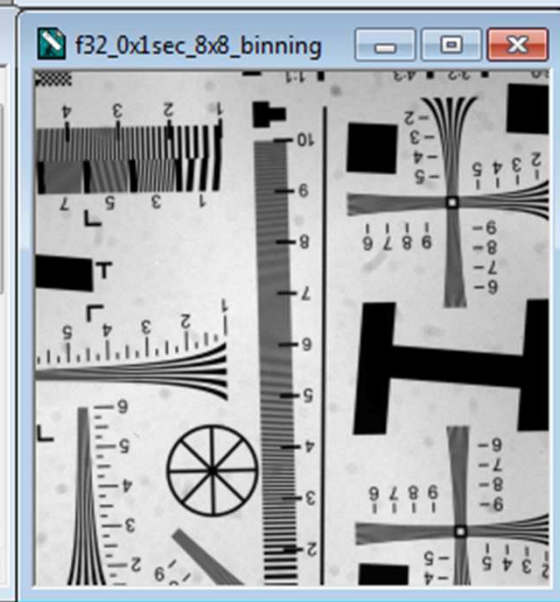14,066 DN
1x1 binning
100% scale
(image cropped)

F8
0.01 seconds
Avg Signal Level
17358DN
2x2 binning
100% scale
(image cropped)

F16
0.01 seconds
Avg Signal Level
17315DN
4x4 binning
100% scale
(image cropped)

F32
0.01 seconds
Avg Signal Level
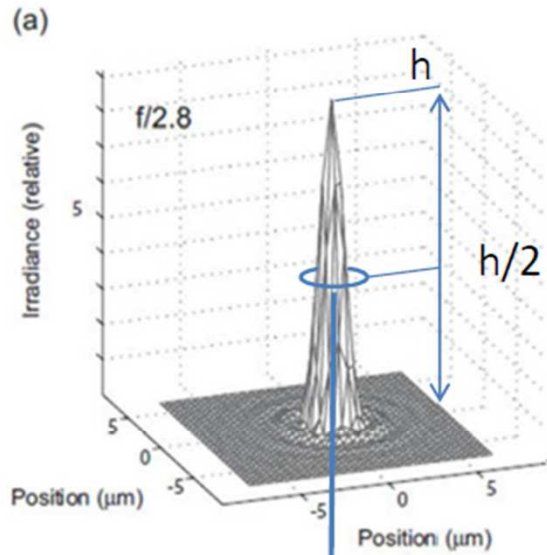18054DN
8x8 binning
100% scale

What about image scale when binning?

- Obviously binning 2x2 increases the size of the "pixel" to 2x what it previously was.
- If that is accompanied by a 2:1 change in F/# then the Nyquist criterion continues to be met for the Diffraction-Limited case
- A 2:1 difference in F/# will make exactly a 2:1 difference in Diffraction-Limited spot size (Airy disk). When coupled with a corresponding adjustment in pixel size, critical sampling is maintained
- It is desirable to bin when adjusting Focal Ratio:
  - Maintains same exposure time
  - Maintains diffraction-limited critical sampling
- Why adjust the focal ratio when using these criteria:
  - Better depth of field using slower F/#
  - But reduced resolution from binning to maintain exposure time and critical sampling

Optics and the Airy Disk:
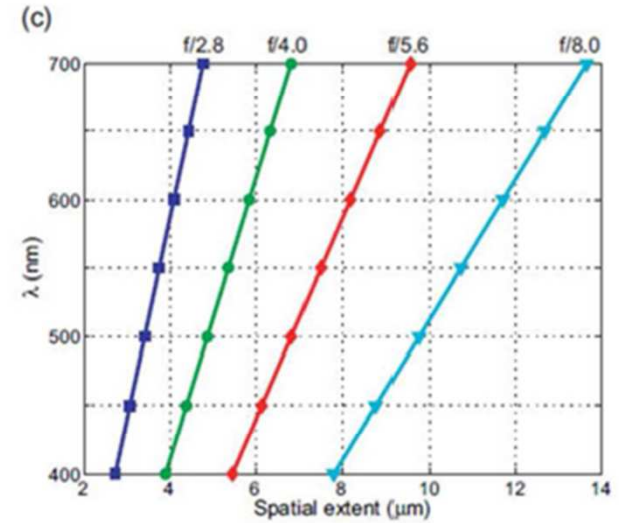## Focal ratio: Sets spot size for diffraction limited optics
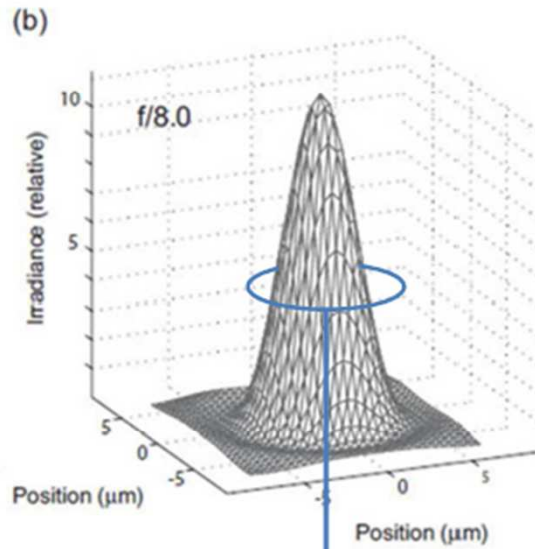
Source: Catrysse

Airy Disk Diameter (microns)

Airy Disk Diameter

~3 microns, f/2.8

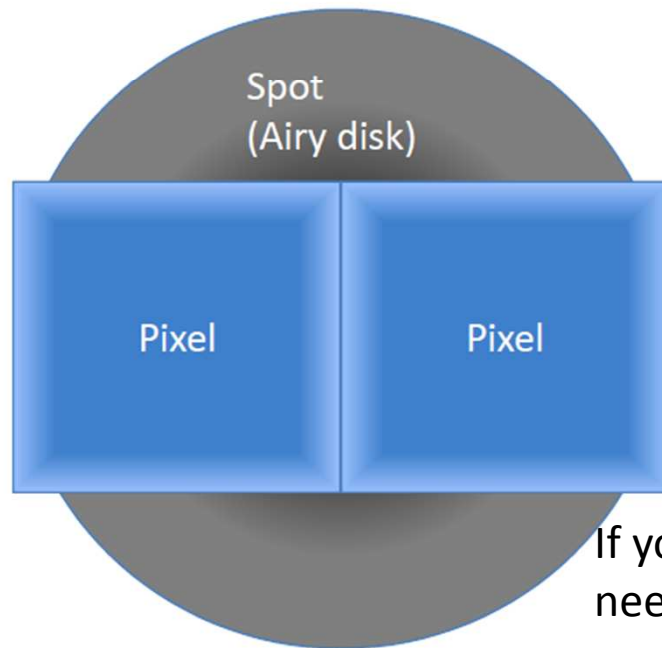~10 microns, f/8.0

If you slow the optics:
need to use a larger pixel

Nyquist Sampling of Airy Disk
Pixel Pitch: Sized to fit Airy Disk (spot):

Spot
(Airy disk)

Pixel | Pixel

Pixel size (microns)	Optimum F#	Airy Diameter (microns)	Optical resolution LP/mm
	550nm test wavelength		
0.75	1.12	1.5	667
0.9	1.34	1.8	556
1	1.49	2	500
1.1	1.64	2.2	455
1.25	1.86	2.5	400
1.4	2.09	2.8	357
1.5	2.24	3	333
1.75	2.61	3.5	286
2	2.98	4	250
2.25	3.35	4.5	222
2.5	3.73	5	200
2.75	4.10	5.5	182
3	4.47	6	167
3.5	5.22	7	143
4	5.96	8	125

If you slow the optics:
need to use a larger pixel

Nyquist Sampling Criteria:
$$Pixel\ size = 1.22 * \lambda * F\#$$

- Exact Nyquist Sampling: 2 pixels to cover Airy Diameter (spot)

For Seeing-Limited spot size, the FWHM of the seeing sets the spot size and should still be covered by two pixels for proper sampling.

Thank you

- I hope this treatise is helpful and clear.
- Feel free to contact me if you have any questions

ML4022 PTC/DTC
Binned 1x1, 2x2

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June 30, 2012

Overview

- A standard PTC / DTC was made on a standard production (circa 2008) FLI ML4022 camera
  - Dark data taken at +15c
  - Light-on data taken at -25C
- Data was taken using 1x1 and 2x2 binning
- Read noise, Full well (saturation) and Kadc were noted
- Binning improves saturation level by 44% while leaving read noise unchanged (9.17 e-)
- 1x1 binning shows the Pixel's photodiode reaching full capacity at saturation (38,000e-)
- 2x2 binning shows the A/D converter reaching full range at saturation (55,800e-). The pixel photodiodes are at 13,500e- when the A/D saturates, 36% of the full well capacity

Noise Versus Signal (DN units)
ML4022
1x1 and 2x2 binned

DTC/PTC by R. Crisp
FLI Microline 4022
June 26, 2012 light off
July 2, 2012 light on
12MHz @ +15c (dark)
12MHz @ -25C (light on)

RN(2x2) =10.4 DN (2X2 binning)
Kadc(2x2) = 0.881 e-/DN
RN(2x2) = 9.18 e-

RN(1x1) = 10.1 DN
Kadc(2x2) = 0.908 e-/DN
RN(1x1) = 9.17 e-

Noise (DN)

- 2x2 binned dark total noise
- 2x2 binned shot noise
- 2x2 binned DFPN
- 2x2 binned FPN
- 2x2 binned total noise
- 1x1 binned dark total noise
- 1x1 binned shot noise
- 1x1 binned DFPN
- 1x1 binned FPN
- 1x1 binned total noise

Dark Fixed Pattern Noise

Slope = +1

Slope = +0.5

Shot noise:
1x1 and 2x2
binning

1x1 binned

2x2 binned

Saturated
1x1 binned

Fixed Pattern Noise

Saturated
2x2 binned

Signal (DN)

-e/DN = 1.54 -e/DN
Dn = 1/0.805 = 124%

Noise Versus Signal (-e)

DTC/PTC by R. Crisp
FLI Microline 4022
June 26, 2012 light off
July 2, 2012 light on
12MHz @ +15c (dark)

Saturated
2x2 binned
(A/D limited)
55.800 e-

- 2x2 binned dark total noise
- 2x2 binned shot noise
- 2x2 binned DFPN
- 2x2 binned total noise
- 2x2 binned FPN
- 1x1 binned dark total noise
- 1x1 binned shot noise
- 1x1 binned DFPN
- 1x1 binned total noise
- 1x1 binned FPN

Noise (-e)

Slope = 1

Slope = +0.5

Slope = 1

Read Noise = 9.17e-

1x1 binned

2x2 binned

Fixed Pattern Noise

Saturated
1x1 binned
(pixel's
photodiode
limited)
38,000 e-

Signal (-e)

Linearity Plot by R. Crisp
FLI Microline 4022
June 26, 2012 light off
12MHz @ +15c (Dark)
12MHz @ -25C (Light)
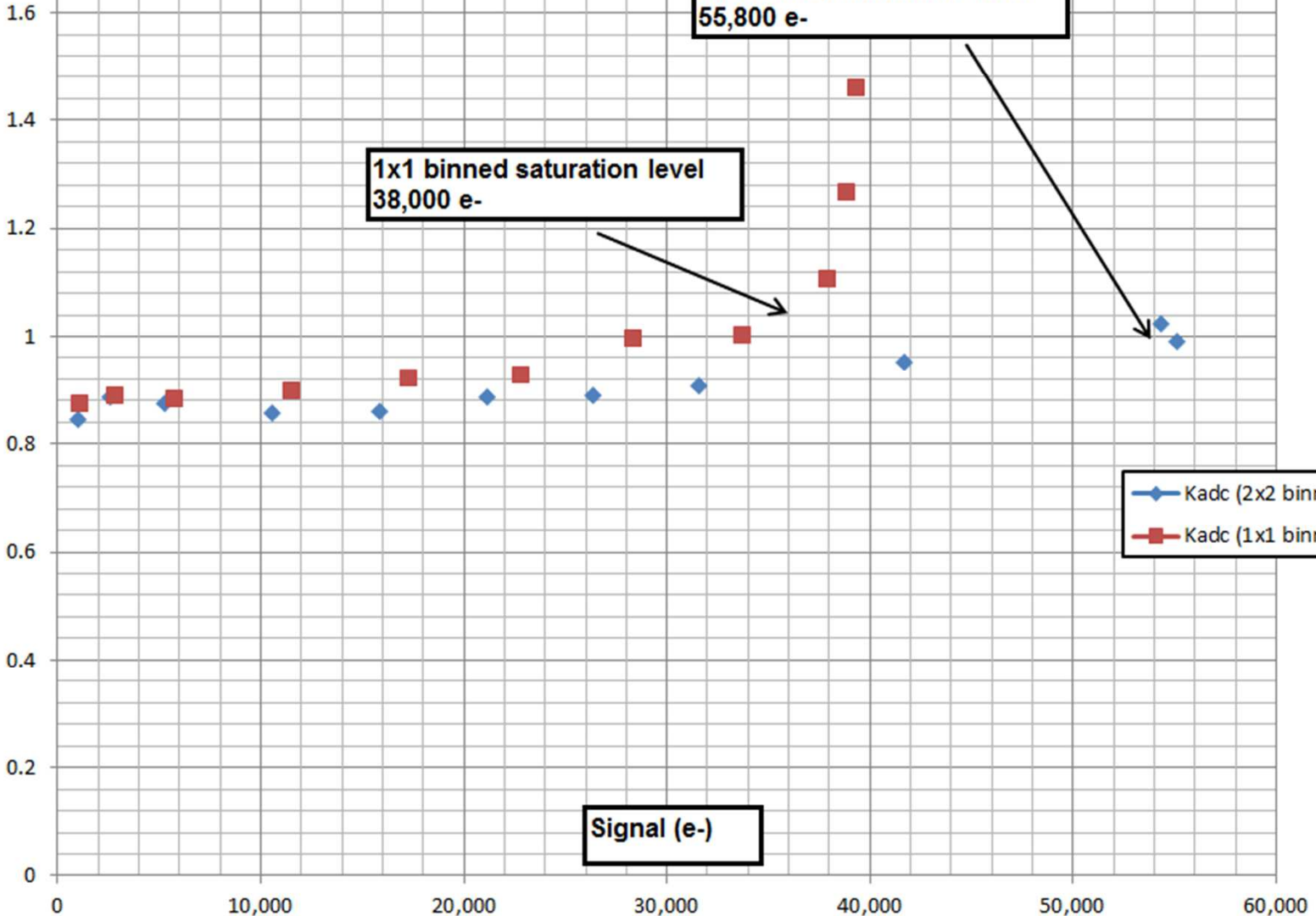
Kadc vs Signal (e-)
ML 4022
2x2 and 1x1 binned

2x2 binned saturation level
55,800 e-

1x1 binned saturation level
38,000 e-

Kadc (e-/DN)

Signal (e-)

Kadc (2x2 binning)
Kadc (1x1 binning)

Comments about Full Well

- The Full Well Capacity for the 1x1 binned case is lower than for the 2x2 binned case
- Since this is an interline sensor, the pixel's photodiode limits the charge capacity in the non-binned case.
  - The vertical CCD does not saturate: it has higher capacity than does the photodiode to assure complete charge transfer
  - The antiblooming structure prevents pixel-to-pixel image smear upon reaching full well
  - Each Photodiode site has slightly different charge capacity
  - The result is a fixed pattern that never changes once full well is reached (see following page)

Saturation with 1x1 binned Interline
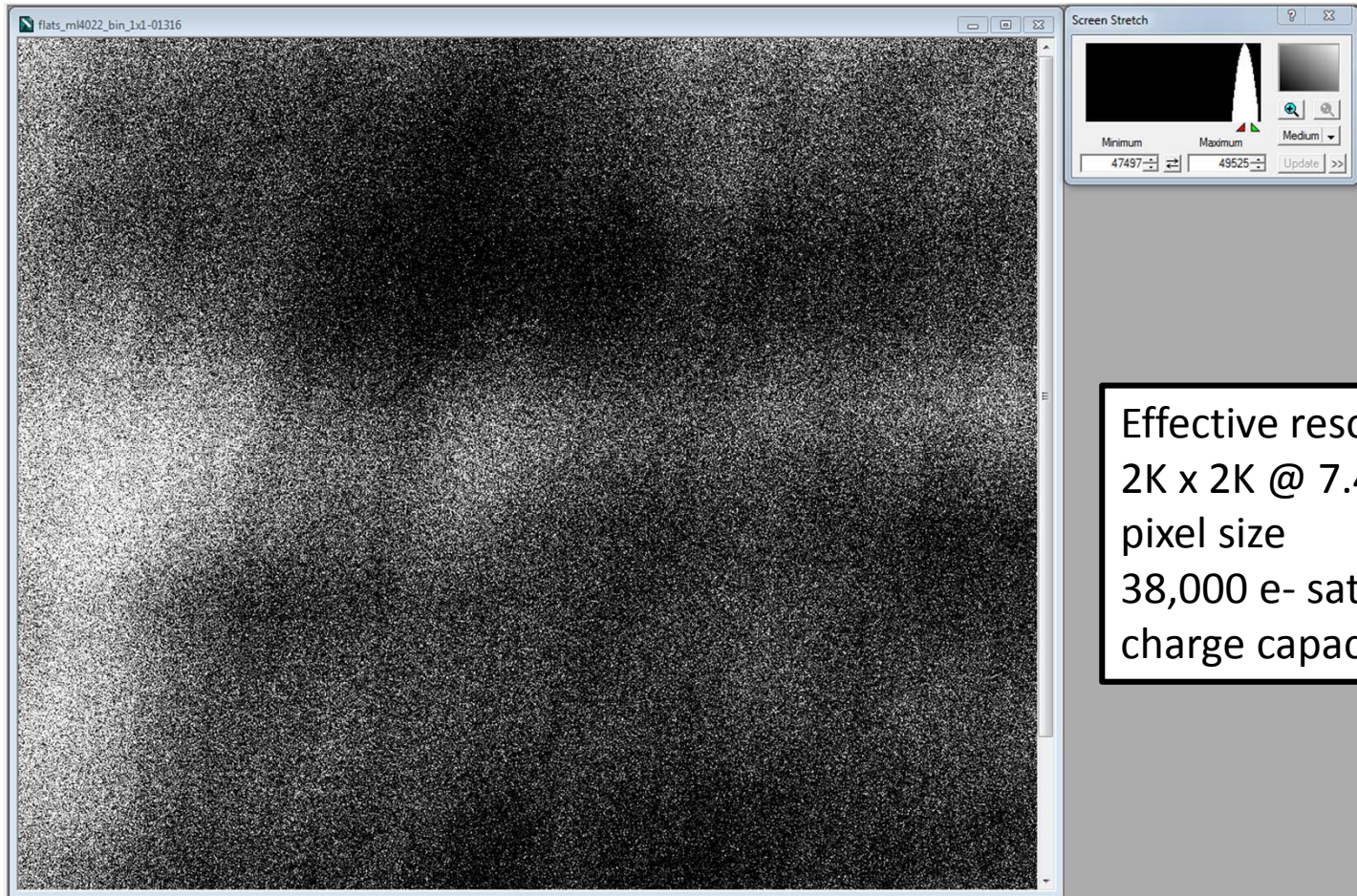
Effective resolution
2K x 2K @ 7.4micron
pixel size
38,000 e- saturation
charge capacity

The pixel's photodiode saturates. The antiblooming structure prevents blooming
The vertical and horizontal readout CCD structures do not saturate
Each Pixel's photodiode has slightly different charge capacity, causing a fixed pattern when
saturated

Full well with 2x2 binning

- The Saturation level for the 2x2 binned case is higher than for the 1x1 binned case
- The A/D converter is what saturates first when operating 2x2 binned
- The all black frame results from all signal levels being larger than can be represented by the A/D converter at full range. DN level for output is 65,535 for all pixels.
- When binned 2x2 the saturation charge capacity is 55,800e-, 44% more than the 38,000e- for the non-binned case.
  - At saturation the pixel's photodiode contains 13,500e-, or 36% of the saturation capacity of the individual photodiode

Saturation with 2x2 binned Interline

Effective resolution
1K x 1K @ 13.6
micron pixel size
55,800e- saturation
charge capacity
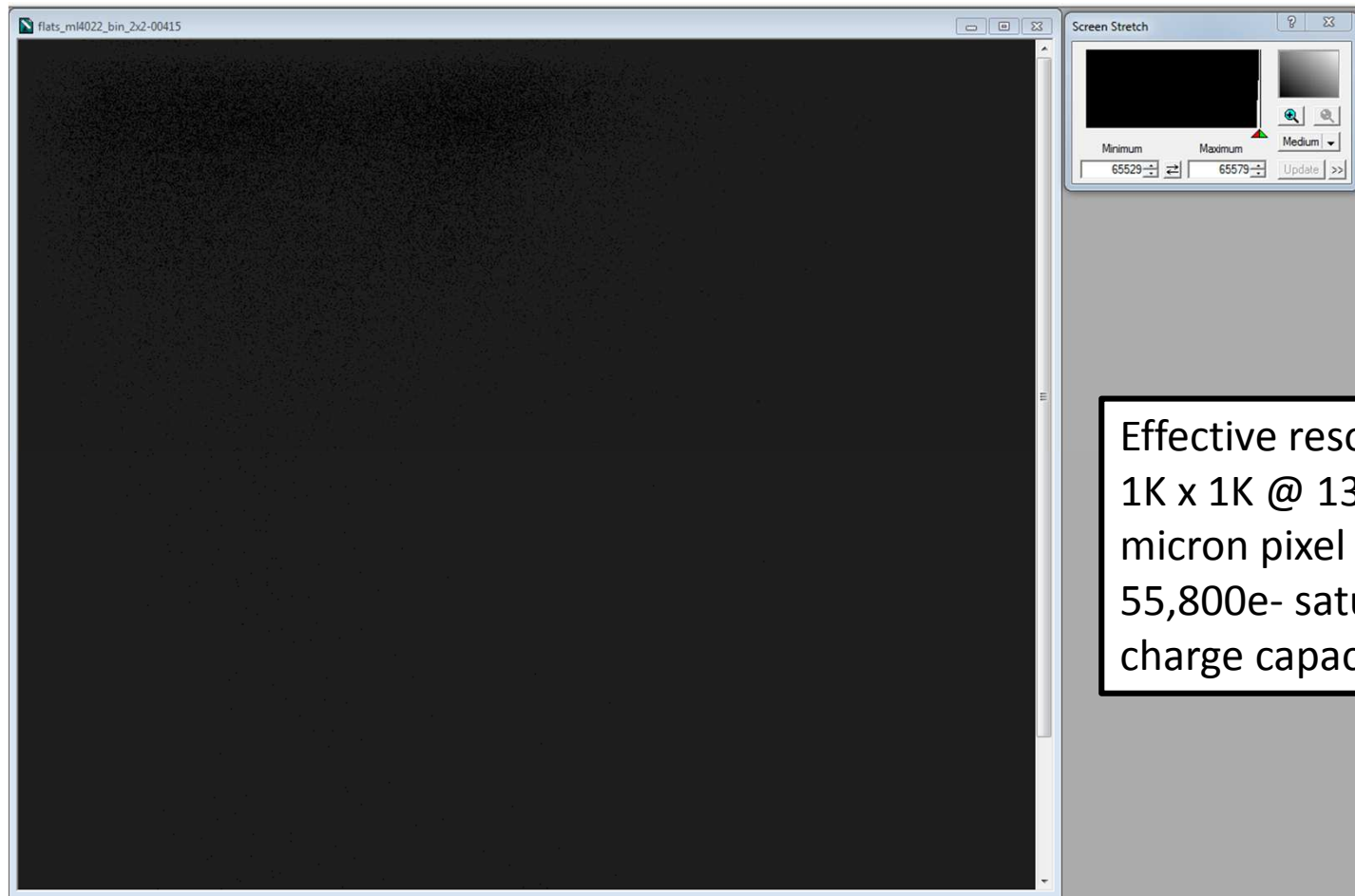
The A/D saturates instead of the pixel's photodiode
Each Pixel's photodiode only holds approximately 36% of full charge capacity when saturation of readout CCD structure is reached
Approximately 44% higher charge capacity results: 55,800e- vs 38,000e-

Summary

ML4022	Binned 1x1	Binned 2x2
Measured Read Noise (e-)	9.17	9.18
Measured Signal Saturation Level (e-)	38,000	55,800
Kadc (e-/DN)	0.900	0.881